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Mechanical quasi-equilibrium and thermovibrational convective instability in an inclined fluid layer

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Abstract—The linear stability of mechanical quasi-equilibrium of a long inclined plane fluid layer, in the presence of a constant temperature gradient, subject to a static gravity field and high frequency vibration is investigated theoretically. The layer is oriented in an arbitrary respect to the vertical. The boundaries of the layer are assumed to be rigid and highly conducting. Each of two vectors—the temperature gradient and the axis of vibration—can have one of the four orientations: vertical (v), longitudinal (l), horizontal (h), and transversal (t). Thus a total of sixteen situations are studied. The consideration is based on the equations system describing mean (averaged) fields in the frame of an averaging method. The possibility and necessary conditions of mechanical quasi-equilibrium existance are studied. The spectral amplitude problem for small two-dimensional normal disturbances is formulated. In the case of long-wave instability, the spectral problem is solved asymptotically using the wave number as a small parameter for expansion. In the case of an arbitrary value of wave number, the spectral problem is solved numerically. The boundaries of stability and critical disturbance characteristics are determined for all the sixteen cases mentioned before.

1. INTRODUCTION

It is known that the vibration of a cavity filled with fluid in the presence of temperature inhomogeneity may induce regular averaged flows, even when static gravity is absent, i.e. in the state of weightlessness (the phenomenon of thermovibrational convection) [1, 2]. In the limiting case of high frequency and small amplitude of vibration, the method of averaging may be applied for the analysis which is often used in different branches of physics and mechanics (see ref. [3]). The method results in receiving a closed equation system describing the behaviour of average fields of velocity, temperature and pressure. In thermal convection theory the method of averaging was first developed in ref. [4], where the effect of high frequency vertical vibration on the convective stability of horizontal fluid layer was studied.

Under certain conditions the mechanical quasi-equilibrium is possible, i.e. the state at which the averaged velocity is absent but pulsational component exists, in general. The conditions of mechanical quasi-equilibrium in weightlessness were first ascertained and the stability problem was formulated in refs. [1, 2]. In these papers the stability of plane layer in the presence of transversal temperature gradient and arbitrary orientation of vibration axis was investigated. Some other examples of quasi-equilibrium situations in weightlessness with analysis of linear stability are also presented. In refs. [5–7] the special case of a plane layer in weightlessness for different mutual orientations of temperature gradient and axis of vibration was analysed. The effect of thermal boundary conditions on the stability of plane layer in the presence of transversal temperature gradient and longitudinal vibration is studied in ref. [8].

In the general case when a static gravity field exists, both mechanisms of excitation are operative—thermogravitational and thermovibrational. In ref. [9] a horizontal plane fluid layer with rigid and isothermal boundaries was studied in the presence of a transversal temperature gradient and a high frequency vibrational field with longitudinal vibration axis. The appearance of instability when the system is heated from above, demonstrates the existance of a specific thermovibrational mechanism of excitation. Experimental results presented in ref. [10] are in a good conformity with theoretical ones. Refs. [11–13] are devoted to the study of thermovibrational quasi-equilibrium convective stability of a horizontal fluid layer with internal heat sources; few variants of thermal boundary conditions are discussed. Experimental data [14] confirms theoretical predictions. The special case of a fluid layer with exothermic reaction of the Arrenius-type is considered in ref. [15]. In ref. [16] plane-parallel vibrational convective flows in an inclined fluid layer subject to longitudinal temperature gradient are studied. In some cases mechanical quasi-equilibrium is possible as the result of mutual compensation of gravitational and vibrational volume forces. The stability criterium for this quasi-equilibrium state is determined, but only against long wave disturbances.

NOMENCLATURE

A	temperature gradient
b	amplitude of displacement
D	operator $(d^2/dx^2)-k^2$
$f(x)$	amplitude of F-disturbance
F	stream-function for $\mathbf{w}$
g	acceleration of gravity
h	one half of layer thickness
k	wave number
$\mathbf{m}(m_x, 0, m_z)$	unit-vector along temperature gradient
$\mathbf{n}(n_x, 0, n_z)$	unit-vector along axis of vibration
p	pressure
Pr	Prandtl number
Ra	Rayleigh number
Ra_v	vibrational Rayleigh number
t	time
T	temperature
$\mathbf{v}(v_x, v_y, v_z)$	velocity
$\mathbf{w}(w_x, w_y, w_z)$	solenoidal part of vector field $T\mathbf{n}$
(x, y, z)	Cartesian coordinates.

Greek symbols

α	angle of inclination
β	coefficient of thermal expansion
γ	unit-vector along vertical directed upward
Δ	Laplace operator
ε	dimensional vibrational parameter
λ	decrement
$\vartheta(x)$	amplitude of temperature disturbance
$\varkappa$	non-dimensional vibrational parameter
ν	coefficient of kinematic viscosity
χ	coefficient of thermal diffusivity
ρ	reference value of density
ψ	stream-function for $\mathbf{v}$
$\varphi(x)$	amplitude of stream-function disturbance
$\nabla\varphi$	potential part of vector field $T\mathbf{n}$
Ω	angular frequency.

Subscripts

0	equilibrium field
m	extremal value
c	critical value.

Superscripts

$(')$	disturbance.

In our short review we are concerned with only the papers devoted to the study of mechanical quasi-equilibrium convective stability in the presence of high frequency vibrations. Additional bibliography regarding some other aspects of thermovibrational convection can be found in ref. [17].

In the present work we study the mechanical quasi-equilibrium linear convective stability of a plane fluid layer arbitrarily inclined to the vertical. A total of sixteen variants of orientations of equilibrium temperature gradient and vibration axis with respect to the layer are considered, corresponding to four independent orientations of both vectors: vertical (v), longitudinal (l), horizontal (h) and transversal (t). Thus the situation (v, t) means, for example, that the temperature equilibrium gradient is vertical, whereas the axis of vibration is transversal, etc. For each case the possibility and conditions of quasi-equilibrium state are determined, and linear stability analysis is developed. The boundaries of stability for arbitrary normal modes and critical disturbance characteristics are determined.

In Section 2 the statement of the problem is given and the basic equations system for averaged fields is written. Non-dimensional parameters of the problem are listed. The general conditions of mechanical quasi-equilibrium are presented in Section 3. Section 4 contains the stability problem formulation for two-dimensional disturbances of the normal-mode type. The spectral amplitude eigenvalue problem is formulated. The limiting case of long-wave disturbance is considered. In Section 5 the results of quasi-equilibrium and stability analysis are presented and discussed.

2. STATEMENT OF THE PROBLEM. BASIC EQUATIONS

Consider a plane fluid layer bounded by two parallel rigid plates. The layer is inclined with respect to the vertical, the angle of inclination is α. The system of coordinates is shown in Fig. 1. The layer is infinitely long in both directions, y and z. All of the system linearly and harmonically oscillates with a high angular frequency Ω and a small displacement amplitude b in a direction which is characterized by the unit-vector $\mathbf{n}$. We consider the situations, when inhomogeneity of temperature exists; the conditions of heating will be formulated in concrete form later.

In the presence of a static gravity field and vibration, the convection is caused by two different mechanisms—thermogravitational and thermovibrational. We shall use for analysis the equation system governing the behaviour of averaged fields of velocity, temperature and convective pressure. This system is derived from standard Boussinesq equations

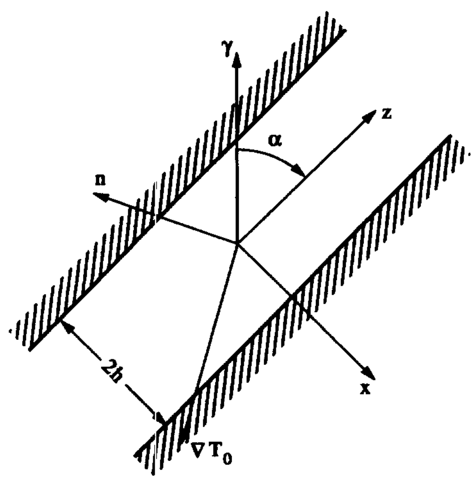

Fig. 1. Geometry of the problem and coordinate system.

for free thermal convection, with the help of averaging method, and can be written in the form

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p+\nu\Delta\mathbf{v}+g\beta T\boldsymbol{\gamma} +\varepsilon(\mathbf{w}\nabla)(T\mathbf{n}-\mathbf{w}) \quad (1)$$

$$\frac{\partial T}{\partial t}+\mathbf{v}\nabla T = \chi\Delta T \quad (2)$$

$$\operatorname{div}\mathbf{v} = 0 \quad (3)$$

$$\operatorname{div}\mathbf{w} = 0 \quad \operatorname{rot}\mathbf{w} = \nabla T\times\mathbf{n}. \quad (4)$$

Here $\mathbf{v}$, T, p are averaged fields of velocity, temperature and pressure—slowly varying with time variables; $\mathbf{w}$ is an additionally "slow" variable, it is the solenoidal part of vector-field $T\mathbf{n}$: $T\mathbf{n} = \mathbf{w}+\nabla\varphi$ ($\nabla\varphi$ is potential part of $T\mathbf{n}$), on the other hand, $\mathbf{w}$ is proportional to the amplitude of oscillatory velocity component; $\boldsymbol{\gamma}(-\sin\alpha, 0, \cos\alpha)$ is the unit-vector directed vertically upward; $\mathbf{n}(n_x, 0, n_z)$ is the unit-vector directed along the vibration axis; β, ν, χ, are the coefficients of thermal expansion, kinematic viscosity and heat diffusivity, respectively; ρ is the reference value of density; ε is dimensional vibrational parameter, appearing in the limiting case of high frequency Ω and small displacement amplitude b in the frame of averaging approach: $\varepsilon = \frac{1}{2}(\beta b\Omega)^2$.

The boundaries of the layer $x = \pm h$ are assumed to be rigid. So the non-slip conditions for mean velocity and non-overflowing for an oscillatory component should be imposed: $x = \pm h$: $\mathbf{v} = 0$, $w_x = 0$. As for boundary conditions for temperature, note that hereafter we shall consider, in the main, the situations when quasi-equilibrium is possible. So the temperature must be distributed on the boundaries in such a manner as to permit the quasi-equilibrium existence.

The system of equations (1)–(4) may be non-dimensionalized with the help of the following units: h for distance, h^2/ν for time, χ/h for velocity, Ah for temperature gradient and $\mathbf{w}$—field (A is characteristic temperature gradient), $\rho\nu\chi/h^2$ for pressure. A non-dimensional form of the equations may be written, therefore, as

$$\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{Pr}(\mathbf{v}\nabla)\mathbf{v} = -\nabla p+\Delta\mathbf{v}+Ra\cdot T\boldsymbol{\gamma} +Ra_v\cdot(\mathbf{w}\nabla)(T\mathbf{n}-\mathbf{w}) \quad (5)$$

$$Pr\cdot\frac{\partial T}{\partial t}+\mathbf{v}\nabla T = \Delta T \quad (6)$$

$$\operatorname{div}\mathbf{v} = 0 \quad (7)$$

$$\operatorname{div}\mathbf{w} = 0 \quad \operatorname{rot}\mathbf{w} = \nabla T\times\mathbf{n}. \quad (8)$$

The system includes the following set of non-dimensional parameters: the Rayleigh number Ra, the vibrational analog of Rayleigh number Ra_v and the Prandtl number Pr

$$Ra = \frac{g\beta A h^4}{\nu\chi} \quad Ra_v = \frac{(\beta\Omega b A h^2)^2}{2\nu\chi} \quad Pr = \frac{\nu}{\chi}. \quad (9)$$

The additional parameters of the problem are: the angle of inclination α; the components of vector $\mathbf{n}$, n_x and n_z, describing the orientation of vibration axis. In the case of equilibrium, when the temperature gradient is constant, $\nabla T = \mathbf{m}$, two extra parameters appear, m_x and m_z, describing the orientation of equilibrium temperature gradient. Note that Ra_v is positive as a definition.

3. MECHANICAL QUASI-EQUILIBRIUM

Let us now write the conditions of mechanical quasi-equilibrium, i.e. the state at which mean velocity ($\mathbf{v}$) is zero but oscillatory component ($\mathbf{w}$) is not zero, in general. These conditions can be deducted from general systems (5)–(8). Assuming

$$\mathbf{v} = 0 \quad \frac{\partial}{\partial t} = 0 \quad \text{and} \quad p = p_0 \quad T = T_0 \quad \mathbf{w} = \mathbf{w}_0$$

and taking rot-operation of both sides of equation (5), we thus find the necessary conditions of mechanical quasi-equilibrium:

$$Ra\cdot(\nabla T_0\times\boldsymbol{\gamma})+Ra_v\cdot\nabla(\mathbf{w}_0\mathbf{n})\times\nabla T_0 = 0 \quad (10)$$

$$\Delta T = 0 \quad (11)$$

$$\operatorname{div}\mathbf{w}_0 = 0 \quad \operatorname{rot} w_0 = \nabla T_0\times\mathbf{n}. \quad (12)$$

Here T_0 and $\mathbf{w}_0$ are the equilibrium fields of T and $\mathbf{w}$.

We shall discuss the special class of equilibrium when temperature gradient is constant, i.e. $\nabla T_0 = \mathbf{m}$ (in non-dimensional form), $\mathbf{m}$ is the unit-vector directed along the equilibrium temperature gradient. The Laplace equation (11) in this case is satisfied automatically, and we have

$$Ra\cdot(\mathbf{m}\times\boldsymbol{\gamma})+Ra_v\cdot\nabla(\mathbf{w}_0\mathbf{n})\times\mathbf{m} = 0 \quad (13)$$

$$\operatorname{div}\mathbf{w}_0 = 0 \quad \operatorname{rot}\mathbf{w}_0 = \mathbf{m}\times\mathbf{n}. \quad (14)$$

All the vectors, γ, $\mathbf{m}$ and $\mathbf{n}$ are assumed to be belonging to the same vertical plane (x, y). Then assume that in the quasi-equilibrium state the oscillatory component of velocity is longitudinal, i.e. $\mathbf{w}_0(0, 0, w_0)$ where $w_0 = w_0(x)$. So the field $\mathbf{w}_0$ is solenoidal and satisfies the boundary conditions $w_{0x} = 0$ at $x = \pm 1$. The profile of $w_0(x)$ is deduced from equation (14) and is

$$w_0(x) = (m_x n_z - m_z n_x) \cdot x. \tag{15}$$

Note that profile $w_0(x)$ satisfies also the closure condition, i.e. the total oscillatory flux through the layer section is equal to zero

$$\int_{-1}^{+1} w_0(x)\,\mathrm{d}x = 0. \tag{16}$$

Substituting equation (15) into equation (13), we may write 'the hydrostatics equation' in the form

$$Ra \cdot (m_z \sin\alpha + m_x \cos\alpha) + Ra_v \cdot (m_x n_z - m_z n_x) \cdot m_z n_z = 0. \tag{17}$$

Hereafter we shall consider some discrete sets of configurations. Each configuration corresponds to one of the sixteen variants of directions of both vectors $\mathbf{m}$ and $\mathbf{n}$ with respect to the layer: vertical, longitudinal, horizontal and transversal. All the configurations are listed below and shown in Fig. 2:

$$\begin{matrix} (v,v) & (v,l) & (v,h) & (v,t) \\ (l,v) & (l,l) & (l,h) & (l,t) \\ (h,v) & (h,l) & (h,h) & (h,t) \\ (t,v) & (t,l) & (t,h) & (t,t). \end{matrix} \tag{18}$$

The first symbol corresponds to the direction of vector $\mathbf{m}$, the second one to $\mathbf{n}$.

The objective is to study every configuration and to answer the question whether mechanical quasi-equilibrium exists or not. In the case when mechanical quasi-equilibrium is possible, the problem appears to investigate its stability against small disturbances.

4. STABILITY PROBLEM FORMULATION

To study the convective stability of quasi-equilibrium state consider perturbed fields

$$\mathbf{v} \quad T = T_0 + T' \quad p = p_0 + p' \quad \mathbf{w} = \mathbf{w}_0 - \mathbf{w}'. \tag{19}$$

Here v, T', p', w' are small disturbances. After substitution of equation (19) into basic systems (5)–(8) and linearization, we obtain the system of equations for disturbances

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla p' + \Delta \mathbf{v} + Ra T' \gamma + Ra_v \cdot [(\mathbf{w}_0 \nabla)(T' \mathbf{n} - \mathbf{w}') + (\mathbf{w}' \nabla)(T_0 \mathbf{n} - \mathbf{w}_0)] \tag{20}$$

$$Pr \cdot \frac{\partial T}{\partial t} + (\mathbf{v} \cdot \mathbf{m}) = \Delta T' \tag{21}$$

$$\operatorname{div} \mathbf{v} = 0 \tag{22}$$

$$\operatorname{div} \mathbf{w}' = 0 \quad \operatorname{rot} \mathbf{w}' = \nabla \mathbf{T}' \times \mathbf{n}. \tag{23}$$

The boundaries of the layer are assumed to be rigid and highly conducting. So the boundary conditions are

$$x = \pm 1: \quad \mathbf{v} = 0 \quad T' = 0 \quad w'_x = 0. \tag{24}$$

In ref. [2] it was shown that in the case of weightlessness ($Ra = 0$) two-dimensional disturbances are most dangerous. We suppose that there is reason to consider two-dimensional disturbances in our more general case when static gravity field exists. So we consider the disturbances of following structure:

$$\mathbf{v}(v_x, 0, v_z) \quad \mathbf{w}'(w'_x, 0, w'_z) \quad \frac{\partial}{\partial y} = 0$$

and introduce the stream functions ψ and F for the solenoidal fields $\mathbf{v}$ and $\mathbf{w}'$, respectively:

$$v_x = \frac{\partial \psi}{\partial z} \quad v_z = -\frac{\partial \psi}{\partial x}; \quad w'_x = \frac{\partial F}{\partial z} \quad w'_z = -\frac{\partial F}{\partial x}. \tag{25}$$

The system of equations for disturbances in terms of ψ, F and T' can be written in the form

$$\frac{\partial}{\partial t}\Delta\psi = \Delta^2\psi + Ra \cdot \left(\gamma_x \frac{\partial T'}{\partial z} - \gamma_z \frac{\partial T'}{\partial x}\right) - Ra_v \cdot \left[n_z (m_x n_z - m_z n_x) \frac{\partial T'}{\partial z} - m_z n_z \frac{\partial^2 F}{\partial x^2} - m_x n_x \frac{\partial^2 F}{\partial z^2} + (m_x n_z + m_z n_x) \frac{\partial^2 F}{\partial x \partial z} \right] \tag{26}$$

$$Pr \cdot \frac{\partial T'}{\partial t} + \left(m_x \frac{\partial \psi}{\partial z} - m_z \frac{\partial \psi}{\partial x} \right) = \Delta T' \tag{27}$$

$$\Delta F = n_x \frac{\partial T'}{\partial z} - n_z \frac{\partial T'}{\partial x}. \tag{28}$$

Here Δ is the two-dimensional Laplace-operator.

Now introduce the disturbances in the form of normal modes

$$(F, \psi, T') = (f(x), \varphi(x), \vartheta(x)) \cdot \exp(-\lambda t + ikz), \tag{29}$$

where $f(x)$, $\varphi(x)$ and $\vartheta(x)$ are the amplitudes, k is the wave number and λ is the decrement. The substitution of equation (29) into systems (26)–(29) yields a system of linear homogeneous ordinary differential equations

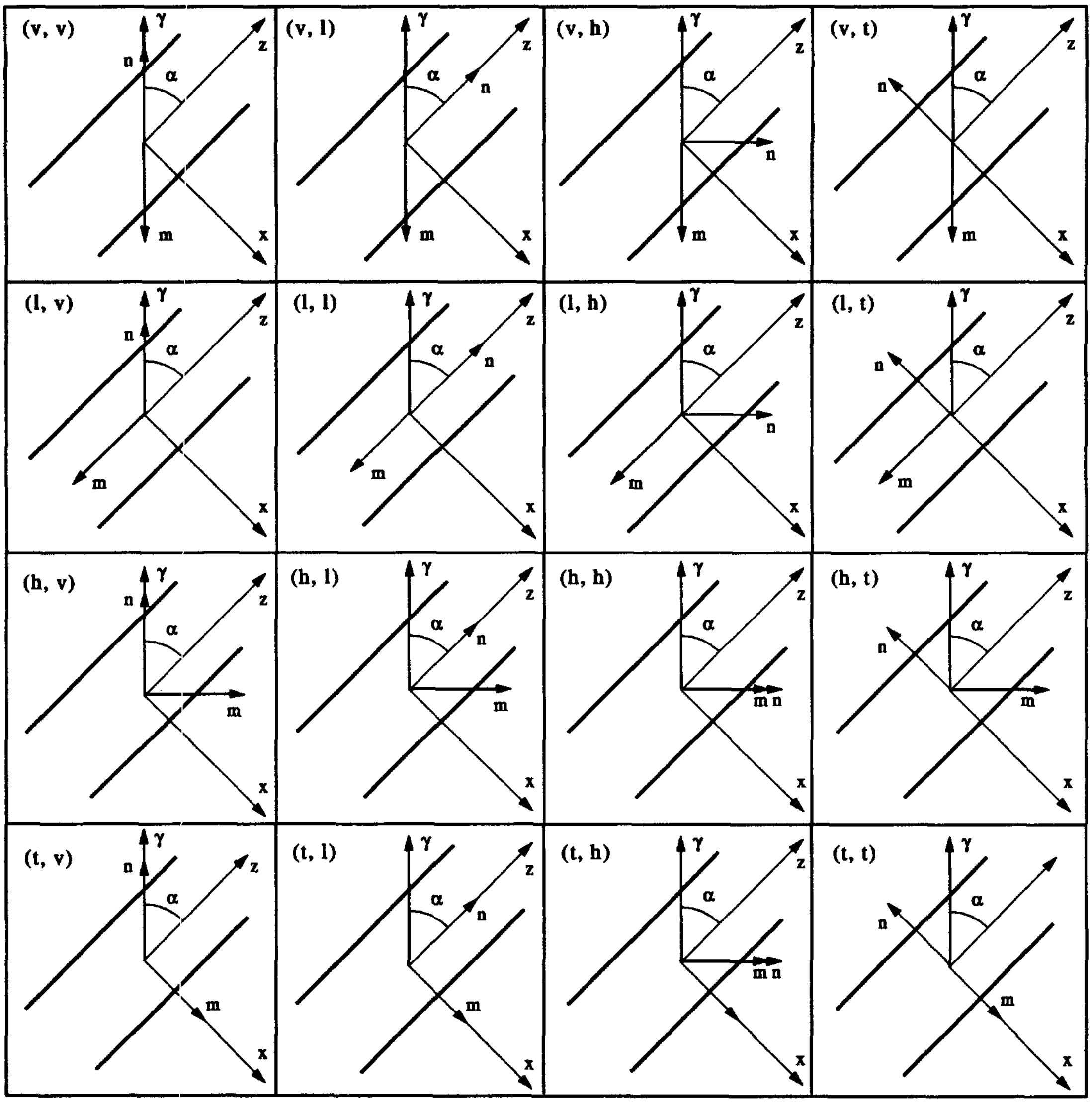

Fig. 2. List of configurations considered.

for amplitudes

$$-\lambda D\varphi = D^2\varphi - Ra(ik\vartheta \sin\alpha + \vartheta' \cos\alpha)$$

$$-Ra_v \cdot [ikn_z(m_x n_z - m_z n_x)\vartheta - m_z n_z f''$$

$$+k^2 m_x n_x f + ik(m_x n_z + m_z n_x) f'] \quad (30)$$

$$-\lambda Pr\vartheta + (ikm_x\varphi - m_z\varphi) = D\vartheta \quad (31)$$

$$Df = ikn_x\vartheta - n_z\vartheta'. \quad (32)$$

Here and hereafter, prime means differentiation respect to transversal coordinate x, and D is the operator

$$D = \frac{d^2}{dx^2} - k^2.$$

The amplitudes must satisfy the following homogeneous boundary conditions:

$$x = \pm 1: \quad \varphi = \varphi' = 0 \quad f = 0 \quad \vartheta = 0. \quad (33)$$

The system of amplitude equations (30)–(32) together with boundary conditions (33) form the spectral amplitude problem with decrement λ as an eigenvalue, depending on all the parameters of the problem. If the decrement is real, then the stability boundary is determined from the condition $\lambda = 0$. In the case when λ is complex, $\lambda = \lambda_r + i\lambda_i$, then the stability boundary can be determined from the condition $\lambda_r = 0$ and λ_i is in this case the neutral frequency of oscillatory disturbance. It is necessary to emphasize that the eigenvalue problem equations (30)–(33) are sensible only in the case when the parameters of the problem are connected by the relation (17), which is the condition of quasi-equilibrium existance.

It is known that in the case when vibration is absent ($Ra_v = 0$) the long-wave mode is most dangerous in some region of the parameters values (see ref. [18]). In this region the neutral curves on the plane (the critical Rayleigh number Ra and the wave number k) have a minimum at the value $k_m = 0$. We may expect that in our case, when a vibration field exists, the long-wave mode will also play an important role.

Substituting $k = 0$ and $\lambda = 0$ into the system equations (30)–(32) we obtain the system of amplitude equations for neutral monotonous long-wave disturbances

$$\varphi^{iv} - (Ra \cdot \cos\alpha - Ra_v \cdot m_z \cdot n_z^2) \cdot \vartheta' = 0 \quad (34)$$

$$m_z \varphi' + \vartheta'' = 0. \quad (35)$$

The boundary conditions are

$$x = \pm 1: \quad \varphi = \varphi' = 0 \quad \vartheta = 0. \quad (36)$$

The solution of the eigenvalue problem equations (34)–(35) can be written in the form (the first level of the spectrum)

$$\varphi = \cos\pi x + 1 \quad \vartheta = -\frac{m_z}{\pi}\sin\pi x. \quad (37)$$

The condition of non-trivial solution existence leads to the stability boundary against long-wave disturbances

$$Ra \cdot \cos\alpha + Ra_v \cdot m_z \cdot n_z^2 = -\frac{\pi^4}{m_z}. \quad (38)$$

The stability boundary determined by this relation corresponds to the mode with $k = 0$. To find out whether this mode is very dangerous or not, it is necessary to compare with the results obtained for the case of finite k. The solution of the complete amplitude problem equations (30)–(33) for the case of arbitrary wave number was found numerically. The Runge–Kutta method of straightforward step-by-step numerical integration was used in combination with a shooting procedure.

5. RESULTS AND DISCUSSION

Case (v, v)

We begin with the case when both vectors, the temperature gradient and the axis of vibration, are vertical. In this case $m_x = \sin\alpha$, $m_z = -\cos\alpha$, $n_x = -\sin\alpha$ and $n_z = \cos\alpha$. As we can see, equation (15) then leads to $\mathbf{w}_0 = 0$. Thus we have to deal with the case of real equilibrium, namely with the state when both averaged and oscillatory components of velocity are equal to zero. The inspection of 'hydrostatics equation' (13) or (17) shows that mechanical equilibrium is possible at arbitrary values of both regime parameters, Ra and Ra_v, since $[\mathbf{m} \times \gamma] = 0$ and $\mathbf{w}_0 = 0$.

In the absence of vibration, $Ra_v = 0$, the problem reduces to the one which describes the stability of equilibrium in an inclined fluid layer in a static gravity field when heated from below. The solution of this problem is known (see ref. [18]). In the vertical layer the threshold of convertion is connected with long-wave disturbances evolution. The long-wave mode is still very dangerous when the angle of inclination is small: $\alpha < \alpha_0$ where α_0 is a critical value which is $\alpha_0 \approx 21°$. If α exceeds the critical value α_0 the transition to cellular node with $k_m \neq 0$ takes place. In the limiting case $\alpha \to 90°$ we obtain the classical Rayleigh–Benard problem on the stability of a plane horizontal layer heated from below. The critical parameters of instability are: the minimal critical Rayleigh number is $Ra_m = 106.7$ and the critical wave number is $k_m = 1.56$ (recall that in this paper one half of the layer thickness is chosen as a unit for distance).

Now consider the effect of high frequency vibrations on the equilibrium stability.

In Fig. 3 the families of neutral curves in the plane (critical Rayleigh number Ra—wave number k) for two angles of inclination and a few values of vibrational Rayleigh number Ra_v are presented. As can be seen, in the region of small angles of inclination (Fig. 3a), the boundary of stability is connected with long-wave modes if Ra_v is relatively small (the neutral curves have minimums at $k_m = 0$), whereas in the region of large Ra_v the instability is caused by cellular modes (the minimums correspond to finite $k_m \neq 0$). The situation is different if $\alpha = 50°$ (Fig. 3b). In this case the instability is of cellular character in the region of relatively small Ra_v and becomes a long-wave one when Ra_v is large enough. The calculations show that when the orientation of the layer is close to horizontal, the cellular character of instability is kept up to very large values of Ra_v.

The summary results are given in Fig. 4, where the critical Rayleigh number Ra_m (minimized with respect to k) and critical wave number k_m are presented as the functions of inclination angle α for few values of Ra_v. From Fig. 4a it is seen that in all the interval of the inclination angle, $0° \leqslant \alpha \leqslant 90°$, the critical value Ra_m increases monotonously as far as Ra_v increases (stabilization). In the region of relatively small Ra_v the instability is of long-wave form when the orientation of the layer is close to vertical one and transition to cellular form takes place when α increases (Fig. 4b). If Ra_v is large enough there is the region of long-wave instability at intermediate angles of inclination α. The boundaries of stability are obtained numerically, but in the region where long-wave modes are most dangerous and responsible for instability, there is a good conformity with the formula (35) which is in our case

$$Ra = \frac{\pi^4}{\cos^2\alpha} + Ra_v \cdot \cos^2\alpha. \quad (39)$$

Finally we present the critical values of Ra_m as functions of Ra_v for two limiting cases of the layer orien-

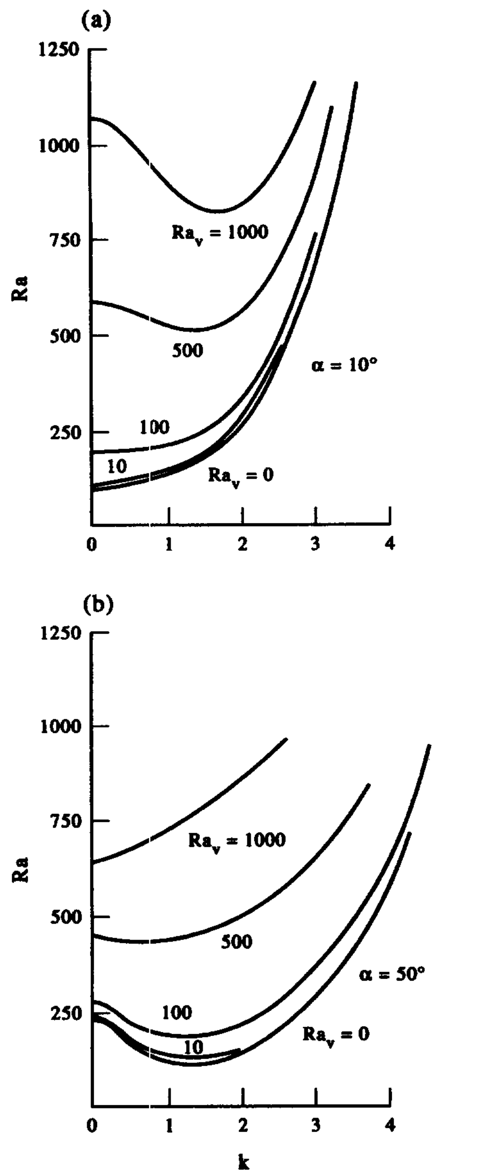

Fig. 3. Neutral curves for different values of Ra_v; (v, v); (a) $\alpha = 10°$, (b) $\alpha = 50°$.

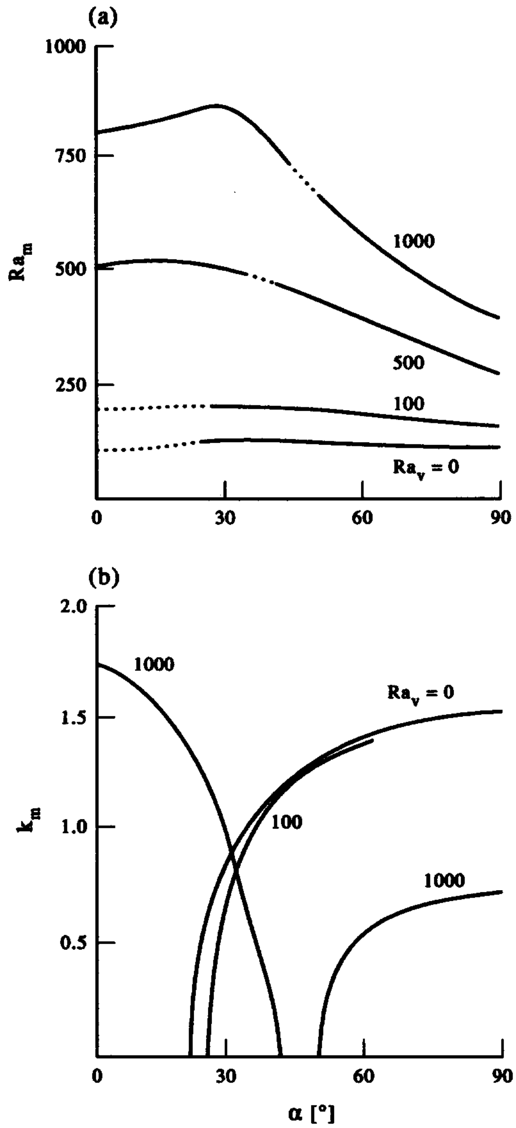

Fig. 4. Critical instability parameters as functions of inclination angle; (v, v); (a) critical Rayleigh numbers (dotted parts of the curves correspond to long-wave mode), (b) critical wave numbers.

tation, vertical ($\alpha = 0°$) and horizontal ($\alpha = 90°$) (Fig. 5). The calculations show that in the region of large values of Ra_v for $\alpha = 90°$ the asymptotics takes place

$$Ra_m = 11.4 \cdot (Ra_v)^{1/2}. \quad (40)$$

This relation means that it is possible to introduce the non-dimensional vibrational parameter

$$\varkappa = \frac{b\Omega}{gh^2}\sqrt{\nu\chi}$$

which is not dependent on temperature gradient. If $\varkappa < \varkappa_0 = 0.124$ then instability takes place in the region of large Ra; if $\varkappa > \varkappa_0$ the equilibrium is absolutely stable (see ref. [18]). This theoretical prediction was confirmed experimentally in ref. [10]. The experimental value of the critical parameter is $\varkappa_0 = 0.16$. The discrepancy is maybe caused by the hard character of convection excitation which was observed in the experiments.

We may summarize that in the case considered the specific thermovibrational mechanism of excitation is not operative. The effect of high frequency vibrations is only a stabilizing one.

Case (v, 1)

Now consider the case at which the quasi-equilibrium temperature gradient is vertical, as in the previous case (v, v), but the axis of vibration is longitudinal, i.e. $m_x = \sin\alpha$, $m_z = -\cos\alpha$, $n_x = 0$, $n_z = 1$

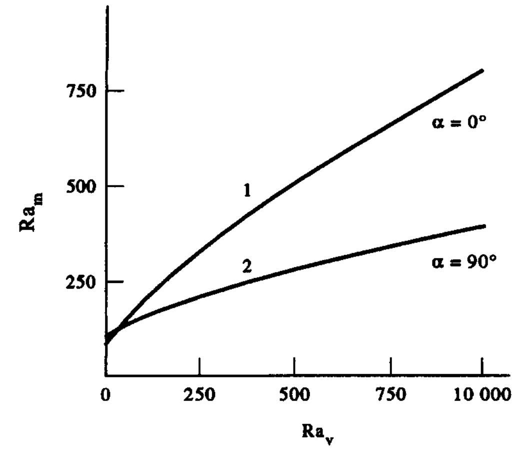

Fig. 5. Critical Rayleigh number as a function of Ra_v for two limiting orientations of the layer; (v, v); 1—vertical orientation ($\alpha = 0°$), 2—horizontal orientation ($\alpha = 90°$).

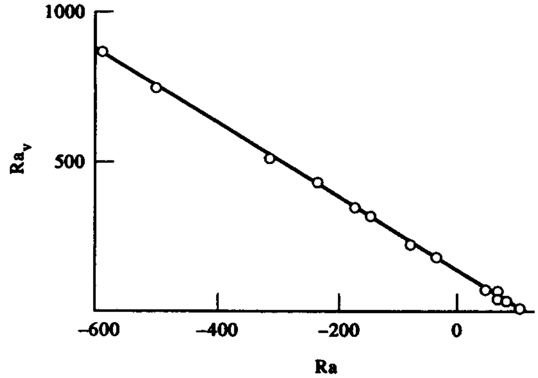

Fig. 6. The boundary of stability; (v, l); $\alpha = 90°$. Solid curve—theory [9], points—experiment [10].

and $w_0 = \sin\alpha \cdot x$. Since $[\mathbf{m} \times \gamma] = 0$ the hydrostatics equation (17) yields

$$Ra_v \cdot \sin\alpha \cdot \cos\alpha = 0. \tag{41}$$

If $Ra_v = 0$ (vibration is absent) then the equilibrium in the static gravity field exists at an arbitrary value of the inclination angle; the result of stability analysis is presented in Fig. 4a, curve $Ra_v = 0$. If both the parameters Ra and Ra_v have arbitrary values then there are only two limiting orientations, $\alpha = 0°$ and $\alpha = 90°$, when quasi-equilibrium is possible. The case $\alpha = 0°$ (vertical layer in the presence of longitudinal temperature gradient and axis of vibration) is already considered for in this case the configurations (v, v) and (v, l) coincide. The critical values Ra are shown in Fig. 5, curve 1.

The configuration $\alpha = 90°$ (horizontal layer in the presence of transversal temperature gradient and longitudinal axis of vibration) is maybe one of the most interesting, because in this configuration the active role of the thermovibrational instability mechanism is distinctly revealed. The theoretical results were obtained in ref. [9] and confirmed experimentally in ref. [10]. We present here the results for completeness in Fig. 6. The boundary of stability in the plane (Ra, Ra_v) is almost a straight line. The point corresponding to $Ra_v = 0$ gives the boundary of the Rayleigh–Benard convective stability of a horizontal layer heated from below. The point $Ra = 0$ corresponds to the boundary of thermovibrational convective stability in weightlessness. The critical parameters are $Ra_{vm} = 133.1$ and $k_m = 1.61$, or if we choose the total thickness of the layer as the unit of distance, then $Ra_{vm} = 2129$ and $k_m = 3,23$ (see ref. [1]). The existence of instability in the region $Ra < 0$ corresponding to the heating from above, is the straight consequence of the operative activity of thermovibrational excitation mechanism.

It is possibly to show that in the case of a horizontal orientation ($\alpha = 90°$) in the presence of a transversal temperature gradient the quasi-equilibrium exists even at an arbitrary direction of the vibration axis with respect to the layer. The stability curves in the plane (Ra, Ra_v) are presented in Fig. 7 for different values of the inclination angle β of the vibration axis to the horizontal (the minimization with respect to the wave number k is made).

Case (v, h)

This case corresponds to the vertical temperature gradient and horizontal axis of vibration: $m_x = \sin\alpha$, $m_z = -\cos\alpha$, $n_x = \cos\alpha$, $n_z = \sin\alpha$ and $w_0 = x$. Equation (17) in this case reduces to equation (41). As in the case (v, l) there are only two orientations of the layer, vertical ($\alpha = 0°$) and horizontal ($\alpha = 90°$), at which the quasi-equilibrium state is possible for arbitrary values of Ra and Ra_v. The case of $\alpha = 90°$ is already considered; the results are presented in Fig. 6. The case of $\alpha = 0°$, a vertical layer subject to longitudinal temperature gradient and transversal (horizontal) vibrations, is not studied yet. The stability boundary against long-wave mode can be found from

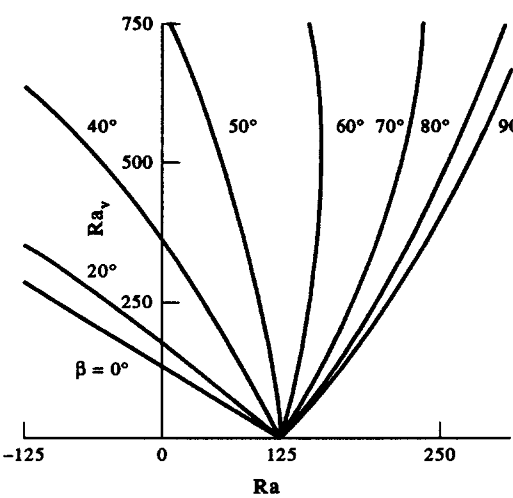

Fig. 7. The boundaries of stability for the case of horizontal layer with transversal temperature gradient; β is an angle between the axis of vibration and horizontal.

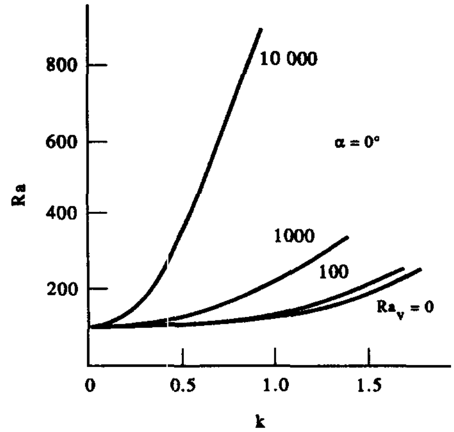

Fig. 8. Neutral curves for different values of Ra_v; (v, h); $\alpha = 0°$.

equation (38). The critical Rayleigh number is $Ra = \pi^4$ and does not depend on Ra_v. The neutral curves for few values of Ra_v are presented in Fig. 8. It is seen that the effect of vibration is a stabilizing one and the long-wave mode is most dangerous.

Case (v, t)

In this case $m_x = \sin\alpha$, $m_z = -\cos\alpha$, $n_x = 1$, $n_z = 0$ and $w_0 = -\cos\alpha \cdot x$; besides $[\mathbf{m} \times \gamma] = 0$ and $(\mathbf{w}_0\mathbf{n}) = 0$. Thus the quasi-equilibrium state is possible at arbitrary values of Ra, Ra_v and α. The results of numerical determination of critical instability characteristics are presented in Figs. 9 and 10. The examples of neutral curves are given in Fig. 9 for two values of inclination angle. The critical minimal Rayleigh numbers and wave numbers are presented in Fig. 10 as functions of α for different values of vibrational Rayleigh number Ra_v. As we can see, for fixed value of Ra_v, instability is caused by long-wave mode in the region $0° \leqslant \alpha \leqslant \alpha_0$. If $\alpha > \alpha_0$ then transition to the cellular form of instability takes place ($k_m \neq 0$). The critical value of the angle inclination is $\alpha_0 \approx 21°$ when $Ra_v = 0$ and increases monotonously as far as Ra_v increases. In the region $0° \leqslant \alpha \leqslant \alpha_0$ formula (38) for minimal critical values of Rayleigh number is valid. So, in our case $Ra_m = \pi^4/\cos^2\alpha$.

Case (l, v)

Now we begin to consider the configurations corresponding to the second line of the set (18). In the case (l, v) we have $m_x = 0$, $m_z = -1$, $n_x = -\sin\alpha$, $n_z = \cos\alpha$ and $w_0 = -\sin\alpha \cdot x$. Equation (17) leads to

$$\sin\alpha(Ra - Ra_v \cdot \cos\alpha) = 0. \qquad (42)$$

The first root of this equation is $\sin\alpha = 0$, i.e. $\alpha = 0°$ and corresponds to the vertical orientation with longitudinal temperature gradient and vibration axis. The equilibrium is possible at arbitrary values of Ra and Ra_v. This case is already studied, see Fig. 5, curve 1.

The second root corresponds to $Ra - Ra_v \cdot \cos\alpha = 0$. That means that quasi-equilibrium is possible at an arbitrary inclination but only if Ra, Ra_v and α are connected by quite definite relation, namely $Ra = Ra_v \cdot \cos\alpha$. For limiting case $\alpha = 0°$ we may find the boundary of stability from Fig. 5, curve 1. If $Ra = Ra_v$ we have the critical condition $Ra = Ra_v = 530$. In the opposite limiting case, $\alpha = 90°$, we obtain $Ra = 0$. Thus, mechanical quasi-equilibrium exists at arbitrary values of Ra_v, but only in the case of pure weightlessness ($Ra = 0$). The analysis performed in ref. [7] shows that the quasi-equilibrium state in weightlessness in the presence of longitudinal temperature gradient and transversal axis of vibration is absolutely stable. So we may expect that critical value $Ra_{vm} \to \infty$ as far as $\alpha \to 90°$.

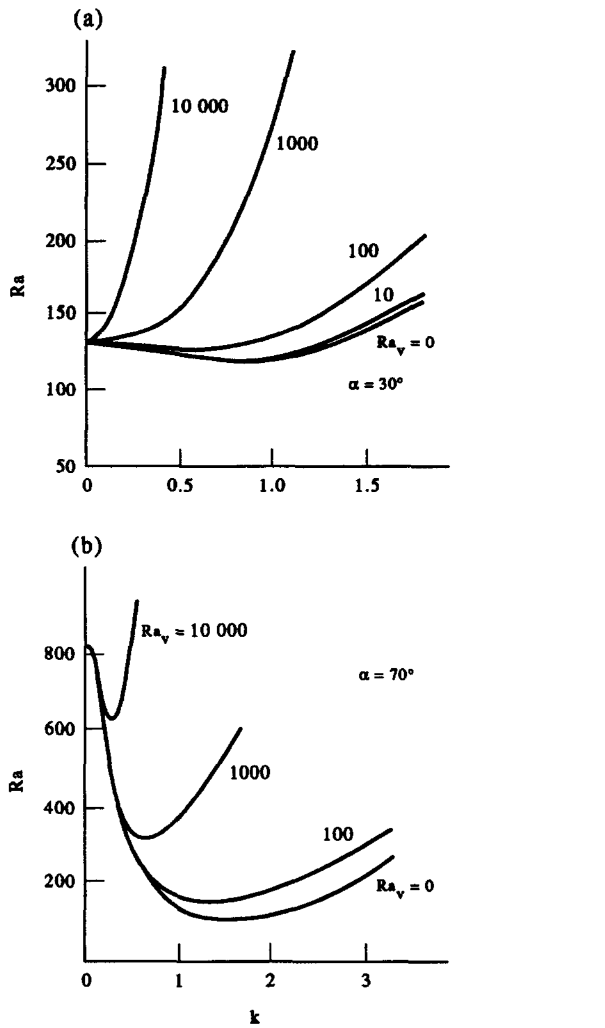

Fig. 9. Neutral curves for different values of Ra_v; (v, t); (a) $\alpha = 30°$, (b) $\alpha = 70°$.

The numerical results are presented in Fig. 11, curves 1. It is seen that in all the interval of inclination angle $0° \leqslant \alpha < 90°$ the instability is caused by cellular mode with the wave number k_m decreasing monotonously when α is increasing.

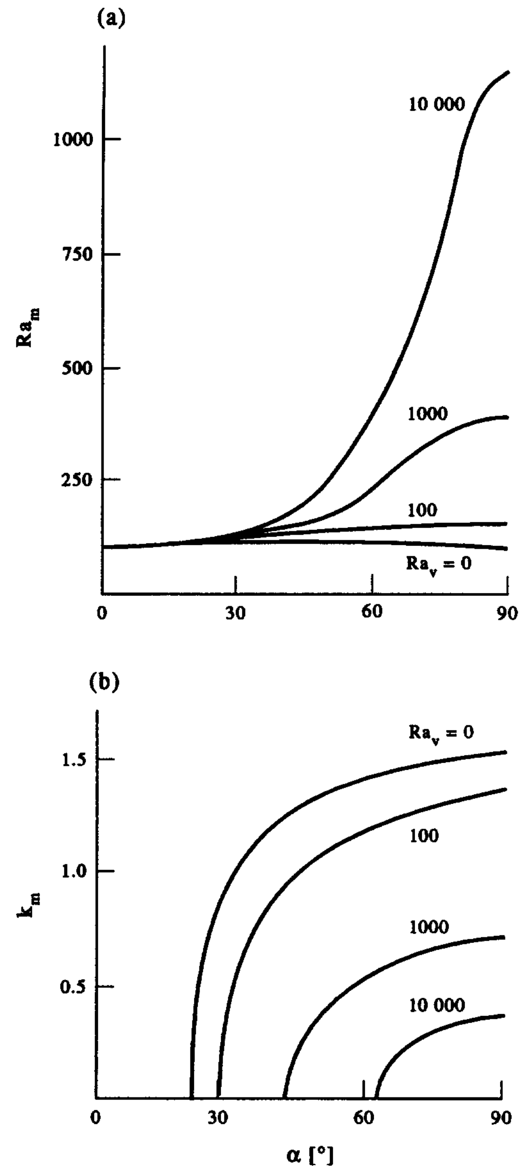

Fig. 10. Critical instability parameters as functions of inclination angle; (v, t); (a) critical Rayleigh numbers, (b) critical wave numbers.

Case (l, l)

Now $m_x = 0$, $m_z = -1$, $n_x = 0$, $n_z = 1$ and $w_0 = 0$. The equilibrium is possible only in the case of vertical orientation, $\alpha = 0°$, at arbitrary Ra and Ra_v. The boundary of stability is described by curve 1, Fig. 5.

Case (l, h)

We have in this case $m_x = 0$, $m_z = -1$, $n_x = \cos\alpha$, $n_z = \sin\alpha$, $w_0 = \cos\alpha \cdot x$ and the conditions of quasi-equilibrium existence in the form $\sin\alpha\ (Ra + Ra_v \cdot \cos\alpha) = 0$.

The first solution is $\alpha = 0°$ and corresponds to a vertical layer subject to longitudinal temperature gradient and transversal vibration; parameters Ra and Ra_v are arbitrary. This case coincides with the

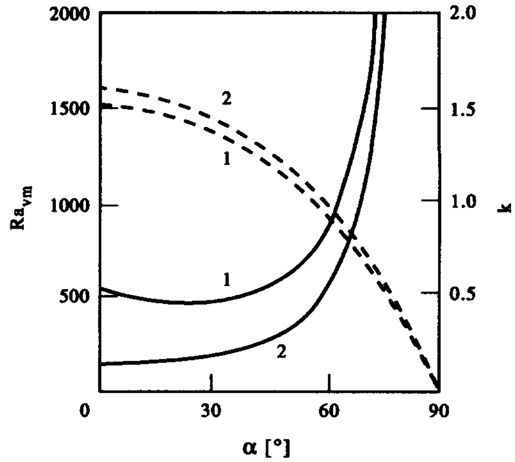

Fig. 11. Critical instability parameters as functions of inclination angle; solid lines—the critical values Ra_{vm}, dashed lines—the critical wave numbers k_m; curves 1—case (l, v) curves 2—case (n, v).

limiting case $\alpha = 0°$ for configuration (v, h); the results of stability study are presented in Fig. 8.

The second solution is $Ra = -Ra_v \cdot \cos\alpha$. This means that if $0° \leqslant \alpha \leqslant 90°$ then the quasi-equilibrium is possible only for $Ra < 0$, i.e. for the heating from above (temperature gradient is directed along the positive z-axis). In the case of $\alpha = 0°$ and negative Ra the quasi-equilibrium is stable (Fig. 8). In the opposite limiting case, $\alpha = 90°$, the quasi-equilibrium exists only in weightlessness, $Ra = 0$. This state corresponds to longitudinal gradient and transversal vibration axis and is also stable. To study the stability in all the interval $0° < \alpha < 90°$, the calculations of characteristic decrements λ were performed. Some results are given in Fig. 12. The fragment of 'lower' part

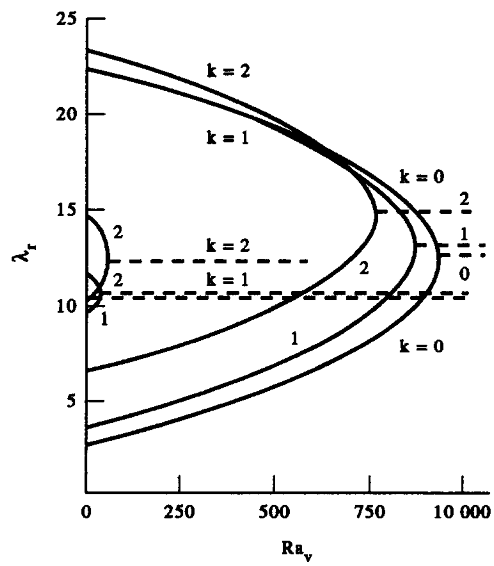

Fig. 12. The fragment of decrement spectrum for $Pr = 1$, $\alpha = 30°$ and few values of k. Solid lines correspond to real λ (monotonous modes), dashed lines—to common real parts λ_r of complex-conjugated pairs (oscillatory modes).

decrement spectrum (four 'lowest' levels) for $Pr = 1$, $\alpha = 30°$ and a few values of wave number is presented as an example. The solid lines corresponds to real values of λ, the dashed ones—to common real parts of the complex—conjugated pairs. As it can be seen, the real parts of all the levels are positive. This means that the quasi-equilibrium states considered are stable. An interesting feature of the picture presented is the merging of monotonous branches to form the pairs of oscillatory disturbances as far as Ra_v increases. Thus in the region of large Ra_v the spectrum consists of damping oscillatory modes.

Case (l, t)

In this case $m_x = 0, m_z = -1, n_x = -1, n_z = 0$ and $w_0 = -x$. The equation of hydrostatics (17) acquires the form $Ra \cdot \sin\alpha = 0$. In the case of weightlessness, $Ra = 0$, we obtain the configuration with a longitudinal temperature gradient and tranversal vibration which is absolutely stable (ref. [7]). If the values of Ra and Ra_v are arbitrary then the mechanical quasi-equilibrium is possible only for the vertical orientation of the layer with vertical temperature gradient and horizontal axis of vibration; the boundaries of stability for this case are shown in Fig. 8.

Case (h, v)

This case corresponds to the horizontal temperature gradient and vertical axis of vibration. Now $m_x = \cos\alpha$, $m_z = \sin\alpha$, $n_x = -\sin\alpha$, $n_z = \cos\alpha$ and $w_0 = x$. Equation (17) leads to

$$Ra + Ra_v \cdot \cos\alpha \cdot \sin\alpha = 0. \tag{43}$$

This relation is a necessary condition for quasi-equilibrium state existence. Let us consider for definiteness of the interval $0° \leqslant \alpha \leqslant 90°$. Then the quasi-equilibrium exists only if Ra is negative (recall that $Ra_v > 0$ as a definition, equation (9)). Case $Ra < 0$ corresponds to the heating from the left side (Fig. 2). In the limiting cases of $\alpha = 0°$ and $\alpha = 90°$, quasi-equilibrium is possible only in weightlessness ($Ra = 0$). Case $\alpha = 90°$ corresponds to an absolutely stable configuration with longitudinal temperature gradient and transversal vibration [7]. In the case $\alpha = 0°$ we obtain a standard problem of thermovibrational convective instability in weightlessness for transversal temperature gradient and longitudinal vibration [1]; the critical parameters are: $Ra_{vm} = 133.1$ and $k_m = 1.61$. The numerical analysis has shown that in all the interval $0° \leqslant \alpha \leqslant 90°$ the instability is caused by cellular mode. The critical value Ra_{vm} is monotonously increasing while the wave number k_m is monotonously decreasing as functions of the inclination angle α; Fig. 11, curves 2.

Case (h, l)

This case is in some sense close to the previous one. We have now $m_x = \cos\alpha$, $m_z = \sin\alpha$, $n_x = 0$, $n_z = 1$ and $w_0 = \cos\alpha \cdot x$. Equation (17) leads to the same relation as in the previous case, namely equation (43). In the limiting case $Ra = 0, \alpha \to 0°$ we obtain the standard stability problem for weightlessness with a transversal temperature gradient and longitudinal axis of vibration. In the case $Ra = 0$ and $\alpha \to 90°$, in contrast to the case of (h, v), we have the configuration with both longitudinal temperature gradient and axis of vibration. This configuration is also absolutely stable. Numerical results are shown in Fig. 13. We see that, as far as the angle of inclination increases, the critical wave number increases too. Thus the instability shift to the side of the short-wave modes there takes place. Also a very sharp quasi-equilibrium stabilization when α increases has to be noted.

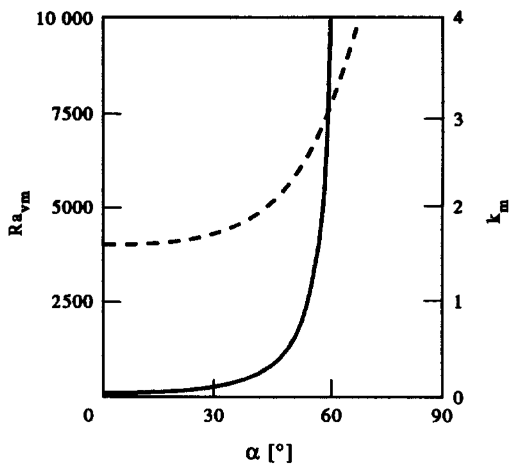

Fig. 13. Critical instability parameters as functions of inclination angle; solid line—the critical value Ra_{vm}, dashed line—the critical wave number k_m; (h, l).

Case (h, h)

This case is degenerate. Indeed, $m_x = n_x = \cos\alpha$, $m_z = n_z = \sin\alpha$, $w_0 = 0$, and we find from equation (17) $Ra = 0$, i.e. the equilibrium is possible only in the case of pure weightlessness at arbitrary Ra_v. But the case at which $Ra = 0$ and the temperature gradient and the axis of vibration are parallel corresponds to the state of absolute stability. That was proved in ref. [2] for the cavity of arbitrary form.

Case (h, t)

This case is also trivial. If $m_x = \cos\alpha$, $m_z = \sin\alpha$, $n_x = -1$ and $n_z = 0$ then $w_0 = \sin\alpha\ x$ and equation (17) leads to $Ra = 0$. Thus, as in the previous case, quasi-equilibrium is possible only in pure weightlessness. We have the configuration at which the vibration axis is transversal, while the quasi-equilibrium temperature gradient is arbitrarily oriented with respect to the layer. This configuration is stable (see ref. [7]).

Case (t, v)

We have $m_x = 1$, $m_z = 0$, $n_x = -\sin\alpha$, $n_z = \cos\alpha$, $w_0 = \cos\alpha \cdot x$ and the equation of hydrostatics (17) has the form

$$Ra \cdot \cos\alpha = 0. \qquad (44)$$

The first root of this equation, $Ra = 0$, corresponds to the case of weightlessness, namely, we obtain the problem of thermovibrational convective instability in a plane fluid layer subject to transversal temperature gradient and arbitrary directed axis of vibration. This problem is solved in ref. [2]. When $\alpha = 0°$, we have the standard configuration with instability parameters $Ra_{vm} = 133.1$ and $k_m = 1.61$. Then Ra_{vm} is monotonously increasing with increase of α (stabilization) and k_m is decreasing. When $\alpha \to 90°$ the following asymptotic formulaes are valid:

$$Ra_{vm} = 2.16 \cdot 10^9 (90° - \alpha)^{-4}, \quad k_m = 0.018 \cdot (90° - \alpha). \qquad (45)$$

Here the angle of inclination α is given in degrees.

The second root of equation (44), $\alpha = 90°$, corresponds to the case of a horizontal layer with transversal temperature gradient and transversal axis of vibration at arbitrary Ra and Ra_v. This case is already discussed, see Fig. 5, curve 2.

Case (t, l)

Now $m_x = 1$, $m_z = 0$, $n_x = 0$, $n_z = 1$, $w_0 = x$ and equation (17) leads to equation (44) as in the previous case. The situation of weightlessness ($Ra = 0$) corresponds to standard configuration with transversal temperature gradient and longitudinal axis of vibration. In the case of $\cos\alpha = 0°$, $\alpha = 90°$, we have now the configuration in a horizontal layer at which both mechanisms of excitation, thermogravitational and thermovibrational, are superimposed, Fig. 6.

Case (t, h)

We have $m_x = 1$, $m_z = 0$, $n_x = \cos\alpha$, $n_z = \sin\alpha$, $w_0 = \sin\alpha \cdot x$ and the equation of hydrostatics in the form of equation (44). The case of weightlessness ($Ra = 0$) coincides with that described in the case of (t, v). In the configuration $\cos\alpha = 0$, $\alpha = 90°$, we obtain the problem of two mechanisms superimposed, Fig. 6.

Case (t, t)

Finally consider the configuration at which both vectors **m** and **n**, are transversal. In this case $m_x = -n_x = 1$, $m_z = n_z = 0$, $w_0 = 0$ and the relation (44) is valid. The mechanical equilibrium state in weightlessness ($Ra = 0$) is now absolutely stable. As for the case $\alpha = 90°$, we obtain the situation at which the vertical vibrations stabilize the normal Rayleigh–Benard instability in the horizontal layer (Fig. 5, curve 2).

6. CONCLUSIONS

The stability of quasi-equilibrium state in an inclined fluid layer in the presence of temperature gradient subject to static gravity field and high vibration is investigated theoretically. The consideration is based on the equations system describing the behaviour of the averaged field. The layer is oriented arbitrarily with respect to the gravity acceleration. In the quasi-equilibrium state the temperature gradient is constant. All three of the vectors, gravity acceleration, temperature gradient and axis of vibration, are assumed to be belonging to the same vertical plane. A total of sixteen configurations are considered depending on orientations of quasi-equilibrium gradient and vibration axis with respect to the layer. For each configuration the possibility of quasi-equilibrium is examined and the problem of linear stability against two-dimensional disturbances of normal-mode-type is studied. The spectral amplitude problems are solved numerically. The stability boundaries and the critical values of the wave numbers of the most dangerous modes are determined. It is shown that in some cases, both of the two mechanisms of instability excitation, thermogravitational and thermovibrational, are superimposed. In other cases the stability effect of high frequency vibrations on the normal thermogravitational instability of Rayleigh–Benard nature takes place.

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